9.4

Sequences

A **sequence is a function** whose domain is the set of positive integers.

A sequence never ends. The numbers in the list are called the **terms** of the sequence.

<u>**Recursive Formulas**</u>: A way of defining a sequence is to assign a value to the first (or the first few) term(s) and specify the nth term by a formula or equation that involves one or more of the terms preceding it. Sequences defined this way are said to be defined **recursively** and the rule or formula is called a **recursive formula.**

Example: $s_1 = 1 \quad s_n = 4s_{n-1}$

$$S_2 = 4 \cdot S_{2-1}$$
$$S_2 = 4 \cdot S_1$$
$$S_2 = 4 \cdot 1 = 4$$

So, the sequence would be 1, 4, 16, 64, …

The Fibonacci sequence: 1,1,2,3,5,… can be defined recursively by:

$$S_1 = 1, \; S_2 = 1$$
$$S_n = S_{n-2} + S_{n-1} \qquad \text{for } n > 2$$

<u>**Limit of a Sequence:**</u> Let $\{a_n\}$ be a sequence of real numbers, and consider $\lim_{n \to \infty} a_n$. If the limit is a finite number L, the sequence ***converges***, and L is the ***limit of the sequence***. If the limit is infinite or nonexistent, the sequence ***diverges***.

Ex.

a) $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n}$

$\lim_{n\to\infty}\frac{1}{n}=0$, so the sequence converges to a limit of 0.

b) $\frac{2}{1},\frac{3}{2},\frac{4}{3},\frac{5}{4},\ldots$

Although the nth term is not explicitly given, we can see that $a_n = \frac{n+1}{n}$.

$\lim_{n\to\infty}\frac{n+1}{n}=\lim_{n\to\infty}(1+\frac{1}{n})=1+0=1$. The sequence converges to a limit of 1.

c) 2, 4, 6, 8, 10, …

This time we can see that a_n = 2n. Since $\lim_{n\to\infty}2n=\infty$, the sequence diverges.

d) -1, 1, -1, 1, …$(-1)^n$…

This sequence oscillates forever between two values and therefore has no limit.

The sequence diverges.

See Example 4 pg. 734.

Arithmetic Sequences

When the difference between successive terms of a sequence is always the same number, the sequence is called **arithmetic.** An **Arithmetic sequence** may be defined **recursively** as:

$a_1 = a$, $\quad a_n - a_{n-1} = d$ or as

$\boldsymbol{a_1 = a}$, $\quad \boldsymbol{a_n = a_{n-1} + d}$ where $a = a_1$ and d are real numbers. (**Recursive Formula**)

The number **a is the first term** and the number **d is called the common difference.**

For an arithmetic sequence { a_n} whose first term is ***a*** and whose common difference is ***d***, the nth term is determined by the **explicit formula**…

$\mathbf{a_n = a + (n\text{-}1)d}$

See Example 5 pg. 734

Geometric Sequences

When the ratio of successive terms of a sequence is always the same nonzero number, the sequence is called **geometric. A geometric sequence** may be defined **recursively** as

$$a_1 = a,\ \frac{a_n}{a_{n-1}} = r \qquad or\ \ as \qquad a_1 = a,\ \ a_n = r \bullet a_{n-1}$$

Where $a_1 = a$ and $r \neq 0$ are real numbers. The number ***a* is the first term,** and the nonzero number **r is called the common ratio**.

The terms of a geometric sequence with first term *a* and common ratio *r* follow the pattern:

$a, ar, ar^2, ar^3 \ldots$

For a geometric sequence {a_n }whose first term is ***a*** and whose common ratio is ***r***, the nth term is determined by the **explicit formula**:

$\boldsymbol{a_n = ar^{n-1},\ \ r \neq 0}$

See Example 6 & 7 pg. 735-736